ECON 1250: Problem Set 1

Due: February 13 (in class or electronically)

1 Base-Rate Neglect

1. Johann comes from a population that is 60% engineers and 40% lawyers. Johann wears a pocket protector (PP). Let p_e be the probability that Johann is an engineer (JE), given that he wears a pocket protector. Let JL denote Johann is a lawyer. Show that $\frac{p_e}{1-p_e} = \frac{0.6Pr(PP|JE)}{0.4Pr(PP|JL)}$ using Bayes' Rule.

2. Suppose instead that Johann comes from a population that is 40% engineers and 60% lawyers. Johann wears a pocket protector (PP). Let p'_e be the probability that Johann is an engineer (JE), given that he wears a pocket protector. Let JL denote Johann is a lawyer. Show that $\frac{p'_e}{1-p'_e} = \frac{0.4Pr(PP|JE)}{0.6Pr(PP|JL)}$ using Bayes' Rule. Also verify that $\frac{\frac{p_e}{1-p_e}}{\frac{p'_e}{1-p'_e}} = \frac{9}{4}$

3. Explain intuitively why the ratio of odds does not depend on $Pr(PP|JE)$

4. What incorrect values of the ratio would imply base-rate neglect?

2 Law of Small Numbers

1. Explain why someone who believes in the Law of Small Numbers would think the sequence of coin flips $THTHHT$ is more likely than the sequence $HHHTTT$ even though the two sequences are equally likely

2. Suppose that basketball players are, during any given game, in one of three states: Hot (they make $\frac{2}{3}$ of their shots), Normal (they make $\frac{1}{2}$ of their shots), or Cold (they make only $\frac{1}{3}$ of their shots). Suppose Dirk Nowitzki is Hot. What is the probability that he will make 4 baskets in a row? What if he is Normal? Cold? If you have no idea what state he'll be in before the game (that is, each state is equally likely), what would you believe about the likelihood that he is Hot after he makes his first 4 baskets in a row?

3. One of the fans at this game believes in the Law of Small Numbers. He has the wrong model of how likely Dirk Nowitzki is to make a basket. Here's how his model works. The fan imagines that there is a deck of 6 cards. When Dirk is Hot, 4 of these cards say 'hit' on them, and only 2 say 'miss.' Every time Dirk takes a shot, one of these cards is drawn randomly without replacement from the deck, and the outcome is whatever the card says. Similarly, when Dirk is Normal or Cold, the outcome of every shot is determined by the draw of a card without replacement from a deck of 6 cards. When Dirk is Normal, the deck has 3 'hit' cards and 3 'miss' cards. When Dirk is Cold, the deck has 2 'hit' card and 4 'miss' cards. Explain how this model corresponds to the Law of Small Numbers (Tversky and Kahneman, 1971; Rabin, 2002). Suppose Dirk Nowitzki is Hot. According to the fan, what is the probability that Dirk will make his first basket? After Dirk makes his first basket, what does the fan believe about the probability that Dirk will make his next basket? Explain why it is lower than the

fan's belief about the probability that Dirk will make his first basket. If the fan has no idea what state Dirk will be in before the game (that is, each state is equally likely), what would he believe about the likelihood that Dirk is Hot after he makes his first 4 baskets in a row? Explain intuitively why the fan's beliefs differ from the normatively correct probability that you calculated.

4. Suppose that, in reality, there is no such thing as being 'Hot' or 'Cold.' Dirk Nowitzki is, in fact, always Normal. Over many games, with what frequencies will Dirk score 0, 1, 2, 3, and 4 baskets in his first 4 attempts? Suppose the fan attends many games and observes these frequencies. Explain why the fan would not believe you if you tried to convince him that there is no such thing as being 'Hot' or 'Cold.'

5. Something similar may be going on in the mutual fund industry. Even if mutual fund returns are almost entirely due to luck, there will be some mutual funds that have done exceptionally well and others that have done exceptionally poorly in recent years due entirely to chance. Explain how the Law of Small Numbers would lead some investors to conclude (falsely) that mutual fund managers differ widely in skill.

3 Prospect Theory

For each of the following, briefly explain (i) why the person's behavior is at first glance inconsistent with expected utility theory, (ii) why it is consistent with prospect theory, and (iii) how the behavior might be reconciled with expected utility theory.

1. Cab drivers in New York City work longer hours on warm, sunny days when their per-hour wage is low.

2. Unionized workers have their wages set 1 year in advance and they receive some bad news that their wages will be cut next year, but they do not cut their spending. However, the previous year when they learned that their wages would increase, they increased their spending.

3. Some students who were about to buy season tickets to a campus theater group were randomly selected and given a discount. During the first part of the season, those who paid full price attended significantly more plays than those who received discounts.

4. People purchase insurance against damage to their telephone wires at 45 cents a month even though the probability that they'd incur the \$60 repair cost in any month is 0.4%.

5. Bettors tend to shift their bets toward longshots (horses with very small chance of winning the race), and away from racetrack favourites (horse with substantial chance of winning the race), later in the racing day.

4 Intertemporal Choice

Otto got one ticket to the four-week long Three Rivers Film Festival, so he can only watch one out of four films. He derives the following utility from these four films:

- Week 1: bad film, $u_1(Film) = 3$

- Week 2: good film, $u_2(Film) = 5$
- Week 3: great film, $u_3(Film) = 8$
- Week 4: Werner Herzog film, $u_4(Film) = 13$

Otto is a (β, δ) quasi-hyperbolic discounter with $\beta = 0.5$ and $\delta = 1$ in this utility function:

$$U_t(u_t, u_{t+1}, ..., u_T) = u_t + \beta \sum_{s=1}^{T-t} \delta^s u_{T+s} \tag{1}$$

- If Otto is naive, that is, if he is unaware of the changes in his preferences and that his future selves will do what the present self thinks they should do, then which film will he watch?
- If Otto is perfectly sophisticated and anticipates his time-inconsistency, what film would he choose to watch?

  Otto's twin, Otto, bought three instead of one ticket. Otherwise, Otto and Otto have the same preferences.
- If Otto is naive, then which film will Otto choose to miss?
- If Otto is perfectly sophisticated, then which film will Otto miss?
- Suppose an economist thinks all people have $\beta = 1$ and wants to estimate δ from a naive agent's behaviour. Find an upper bound on δ such that an exponential discounting agent chooses to miss the same film as the naive agent.

5 Social Preferences

1. Player 1 has the following preference with $1 \geq \beta_1 \geq 0$ and $\alpha_1 \geq \beta_1$

   $$u_1(x) = x_1 - \alpha_1 max\{x_2 - x_1, 0\} - \beta_1 max\{x_1 - x_2, 0\} \tag{2}$$

   Similarly, Player 2 has the following preference with $1 \geq \beta_2 \geq 0$ and $\alpha_2 \geq \beta_2$

   $$u_2(x) = x_2 - \alpha_2 max\{x_1 - x_2, 0\} - \beta_2 max\{x_2 - x_1, 0\} \tag{3}$$

   Player 1 offers a split to Player 2 in the Ultimatum Game and Player 2 decides whether or not to accept or reject the offer. If Player 2 rejects the offer, then both receive zero. Solve for the equilibrium offer $(s, 1-s)$ as a function of the preference parameters $\alpha_1, \beta_1, \alpha_2, \beta_2$. This equilibrium is different depending on whether β_1 is greater than or less than 0.5. (Hint: start by solving for Player 2's best response to Player 1's offer)

2. Consider the Market Game (Roth et al.) in which Players $1, ..., n-1$ make proposals $(s_i, 1-s_i)$. Player n, the responder, then can accept or reject the lowest offer s^L. If she accepts, she gets $1-s^L$ and the winning proposer gets s^L. If several proposers make the low offer, one is selected at random. Suppose all players have Fehr-Schmidt preferences again. Show mathematically and/or explain intuitively why in equilibrium at least two proposers offer $s = 0$ and the responder accepts. Why do Fehr-Schmidt preferences lead to non-zero offers in the Ultimatum Game, but not in the Market Game?

3. There are $n \geq 2$ players in a public goods game who decide simultaneously on their contribution level $g_i \in [0, y]$, where y is each player i's endowment. Player i's payoff is given by $x_i(g_1, ..., g_n) = y - g_i + \eta \sum_{j=1}^{n} g_j$ where $\frac{1}{n} < \eta, 1$. Each player i has Fehr-Schmidt preferences (α_i, β_i)

   - Show that if $\eta + \beta_i < 1$, it is a dominant strategy for player i to choose $g_i = 0$
   - Suppose there are $k < n - 1$ players with $\beta_i = 0$ and $n - k$ players with (α_i, β_i) s.t.

     $$\beta_i \frac{n-1-k}{n-1} - (1 - \eta) > \alpha_i \frac{k}{n-1} \tag{4}$$

     Show that there exists an equilibrium where the $(n - k)$ "fair" players choose $g_i = y$ and the k selfish players choose $g_i = 0$
   - Provide the intuition for Condition (4) above.