EconS 301

Additional Exercises – *Utility Maximization*

Due date: Tuesday, February 4th, 2014, in class.

Exercise #1. Each day Paul, who is in third grade, eats lunch at school. He likes only Twinkies (t) and soda (s), and these provide him a utility of

$$\textbf{Utility} = U(t, s) = \sqrt{ts}$$

a) If Twinkies cost \$0.10 each and soda costs \$0.25 per cup, how should Paul spend the \$1 his mother gives him in order to maximize his utility?

b) If the school tries to discourage Twinkie consumption by raising the price to \$0.40, by how much will Paul's mother have to increase his lunch allowance to provide him with the same level of utility he received in part (a)?

Exercise #2.

a) On a given evening, J.P. enjoys the consumption of cigars (c) and brandy (b) according to the function

$$U(c, b) = 20c - c^2 + 18b - 3b^2$$

How many cigars and glasses of brandy does he consume during an evening? (Cost is no object to J.P.)

b) Lately, however, J.P. has been advised by his doctors that he should limit the sum of glasses of brandy and cigars consumed to 5. How many glasses of brandy and cigars will he consume under these circumstances?

Exercise #3. Mr. A derives utility from martinis *(m)* in proportion to the number he drinks:

$$U(m) = m.$$

Mr. A is very particular about his martinis, however: He only enjoys them made in the exact proportion of two parts gin *(g)* to one part vermouth *(v)*. Hence we can rewrite Mr. A's utility function as

$$U(m) = U(g, v) = min\left(\frac{g}{2}, v\right)$$

a) Graph Mr. A's indifference curve in terms of *g* and *v* for various levels of utility. Show that, regardless of the prices of the two ingredients, Mr. A will never alter the way he mixes martinis.

b) Calculate the demand functions for *g* and *v*.

c) Using the results from part (b), what is Mr. A's indirect utility function?

d) Calculate Mr. A's expenditure function; for each level of utility, show spending as a function of P_g and P_v. *Hint:* Because this problem involves a fixed-proportions utility function, you cannot solve for utility-maximizing decisions by using calculus.

Exercise #4. Suppose that a fast-food junkie derives utility from three goods—soft drinks (*x*), hamburgers (*y*), and ice cream sundaes (*z*)—according to the Cobb-Douglas utility function

$$\boldsymbol{U(x, y, z) = x^{0.5} y^{0.5} (1 + z)^{0.5}}.$$

a) Show that, for $z = 0$, maximization of utility results in the same optimal choices as in the Cobb-Douglas utility function analyzed in class, $\boldsymbol{U(x, y) = x^{0.5} y^{0.5}}$.

b) How do you explain the fact that $z = 0$ is optimal here?

c) How high would this individual's income have to be in order for any *z* to be purchase?